Forecast of omicron wave time evolution

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Adopting an early doubling time of three days for the rate of new infections with the omicron mutant the temporal evolution of the omicron wave in different countries is predicted. The predictions are based on the susceptible-infectious-recovered/removed (SIR) epidemic compartment model with a constant stationary ratio \( k = \mu(t)/a(t) \) between the infection \( a(t) \) and recovery \( \mu(t) \) rate. The fixed early doubling time then uniquely relates the initial infection rate \( a_0 \) to the ratio \( k \), which therefore determines the full temporal evolution of the omicron waves. For each country three scenarios (optimistic, pessimistic, intermediate) are considered and the resulting pandemic parameters are calculated. These include the total number of infected persons, the maximum rate of new infections, the peak time and the maximum 7-day incidence per 100000 persons. Among the considered European countries Denmark has the smallest omicron peak time and the recently observed saturation of the 7-day incidence value at 2478 is in excellent agreement with the prediction in the optimistic scenario. For Germany we predict peak times of the omicron wave ranging from 32 to 38 and 45 days after the start of the omicron wave in the optimistic, intermediate and pessimistic scenario, respectively, corresponding maximum SDI values of 7090, 13263 and 28911, respectively. Adopting Jan 1st, 2022 as the starting date our predictions imply that the maximum of the omicron wave is reached between Feb 1 and Feb 15, 2022. Rather similar values are predicted for Switzerland. Due to an order of magnitude smaller omicron hospitalization rate, due to the high percentage of vaccinated and boosted population, the German health system can cope with maximum omicron SDI value of 2800 which is about a factor 2.5 smaller than the maximum omicron SDI value 7090 in the optimistic case. By either reducing the duration of intensive care during this period of maximum, and/or by making use of the nonuniform spread of the omicron wave across Germany, it seems that the German health system can barely cope with the omicron wave avoiding triage decisions. The reduced omicron hospitalization rate also causes significantly smaller mortality rates compared to the earlier mutants in Germany. In the optimistic scenario one obtains for the total number of fatalities 7445 and for the maximum death rate 418 per day which are about one order of magnitude smaller than the beta fatality rate and total number.

Keywords: coronavirus; extrapolation; omicron mutant; Covid-19

I. INTRODUCTION

After being exposed to several Covid-19 outbursts the recently identified omicron mutant threatens many societies worldwide.\textsuperscript{1,2} Not many details are known so far about its infection characteristics\textsuperscript{3,4} apart from alarming hints (1) that it is spreading at least four times quicker than the beta mutant with a short doubling time of \( t_2 = 3 \) days, and (2) that the existing vaccines, taylored to prevent infections from the earlier alpha, beta, gamma and delta mutants, are less efficient against the vaccines, tailored to prevent infections from the earlier alpha, beta, gamma and delta mutants, respectively. Positively, the omicron mutant seems to lead to average milder symptoms and thus to smaller hospitalization fractions compared to the earlier mutants.

Even with so little details known today it is of high interest to explore quantitatively the future time evolution of the omicron mutant under realistic scenarios of currently taken non-pharmaceutical interventions (NPIs). Of particular interest are reliable estimates of the maximum and total percentage of infected persons from this mutant in order to compare with the available medical capacities in different countries. In the following we provide these estimates by modeling the time evolution of the omicron wave with the susceptible-infectious-recovered/removed (SIR) epidemic compartment model.\textsuperscript{9}

As in our earlier analysis\textsuperscript{10,11} – hereafter referred to as KSSIR-model – we adopt a constant stationary ratio \( k = \mu(t)/a(t) = \text{const.} \) between the infection \( a(t) \) and recovery \( \mu(t) \) rate regulating the transition from susceptible to infected persons and infected to recovered/removed persons in the semi-time case, respectively. As it is so far unclear whether earlier vaccinated persons are not infected by the omicron mutant, we adopt the worst case scenario here and treat the vaccinated persons as fully susceptible to the omicron mutant. However, when calculating hospitalization and mortality rates below we will account for the influence of boosted (with vaccines) persons.

As proven by Table 1 and Figs. 1 and 2 of ref.\textsuperscript{10} the KSSIR-model predicted the temporal evolution of the second wave in several countries convincingly good including the maximum rate \( J_{\text{max}} \) of new infections and the total cumulative number \( J_{\infty} \) of infections as well as the initial and final second wave time dependence and the time of maximum. For the considered countries the maximum deviation in the total

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  \bibitem{rsch@tp4.rub.de} rsch@tp4.rub.de
  \bibitem{mk@mat.ethz.ch} mk@mat.ethz.ch
\end{thebibliography}
number of infected persons is at most 13 percent off from the later recorded values. An outstanding property of the KSSIR-model is that basically only one parameter, the ratio $k$ of recovery and infection rates, fully determines the wave evolution in reduced time $\tau$, whereas the influence of the initial fraction $\eta$ of infected people at the onset of the modeled mutant at time $t_0$ is only minor especially for values of $\eta$ much smaller than unity. Here the reduced time

$$\tau = \int_{t_0}^{t} a(\xi) d\xi$$

(1)
can be calculated for any arbitrary but given real time dependence of the infection rate $a(t)$.

Adopting a constant infection rate $a(t) = a_0$ is a good approximation for rapidly evolving mutant waves and not only for its initial phases, so that in this case the simple relation $\tau = a_0(t - t_0)$ holds between the reduced and the real time. Moreover, by determining the then two decisive parameters $k$ and $a_0$ from the early monitored real time evolution then allows us the accurate determination of all relevant quantities of the considered outburst. The two parameters $k$ and $a_0$ differ among different societies depending besides specific virus mutant properties also on the NPIs taken, the quality and ability of the health care system, and the discipline of the people in keeping distances, wearing masks and following quarantine measures. As the latter are mainly unchanged during different mutant actions it makes sense to relate the $k$ and $a_0$ parameters of the omicron mutant to those of the earlier beta mutant as we will adopt below.

II. RESULTS FROM THE SIR-MODEL

In terms of the reduced time (1) the KSSIR model equations read

$$\frac{dS}{d\tau} = -SI, \quad \frac{dI}{d\tau} = SI - kI, \quad \frac{dR}{d\tau} = kI$$

(2)

obeying the sum constraint

$$S + I + R = 1$$

(3)
at all times. In Eqs. (2)-(3) $S$, $I$ and $R$ denote the fractions of susceptible, infected and recovered/removed persons in a population, respectively, subject to the semi-time initial conditions

$$I(t_0) = I(\tau = 0) = \eta, \quad S(t_0) = S(\tau = 0) = 1 - \eta, \quad R(t_0) = R(\tau = 0) = 0.$$ 

(4)

The rate of new infections and its corresponding cumulative number are given by $j(\tau) = S(\tau)I(\tau)$ and $J(\tau) = \int_0^{\tau} d\xi j(\xi)$, respectively, whereas $\dot{J}(t) = a(t)j(\tau)$ and $J(t) = J(\tau)$.

A. Exact results

In terms of $J$ the exact solution of the KSSIR model in the semi-time case is given by

$$\tau = \int_0^J \frac{dy}{n(y)} = (1 - y)[y + k\epsilon + k\ln(1 - y)]$$

(5)

with $\epsilon = -\ln(1 - \eta)$. The remaining SIR quantities are given by $J(\tau)$ as $S(\tau) = 1 - J(\tau)$, $I(\tau) = J(\tau) + k\eta + k\ln[1 - J(\tau)]$, and $R(\tau) = -k[\epsilon + \ln(1 - J(\tau))]$. Differentiating Eq. (5) with respect to $\tau$ readily yields for the rate of new cases

$$j(\tau) = \frac{dJ}{d\tau} = (1 - J)[J + k\epsilon + k\ln(1 - J)]$$

(6)

As shown before without the explicit inversion of the solution (5) to $J(\tau)$ one obtains for the final cumulative fraction of infected persons

$$J_\infty = \lim_{\tau \to \infty} J(\tau) = 1 + kW_0(\alpha),$$

(7)

with $\alpha = -(1 - \eta)k^{-1}e^{-1/k}$, and for the maximum rate of new infections

$$j_{\text{max}} = (1 - J_0)(1 - J_0 - k) = \frac{k^2}{4}([1 + W_{-1}(\alpha_0)]^2 - 1),$$

(8)

occurring at

$$J_0 = 1 + \frac{k^2}{2}W_{-1}(\alpha_0), \quad \alpha_0 = \frac{2\alpha}{e}$$

(9)
in terms of the principal ($W_0$) and non-principal ($W_{-1}$) solution of Lambert’s equation, the well-known and documented Lambert functions. We emphasize that for small values of $\eta \ll 1$ the results (7) and (9) are basically independent of the value of $\eta$ and only determined by the parameter $k$. The first Eq. (8) implies

$$J_0(k) = 1 - \frac{k}{2} - \sqrt{\left(\frac{k}{2}\right)^2 + j_{\text{max}}}$$

(10)

B. Approximate results

Very accurate approximations have been obtained for

$$j_{\text{max}}(k) \approx \frac{(1 - k)^2(7 + 8k)}{14(2 - k)(1 + k)},$$

$$J_\infty(k) \approx \frac{7 + k - 8k^2}{7},$$

(11)

so that Eq. (10) provides the approximation

$$J_0(k) \approx 1 - \frac{k}{2} - \sqrt{\left(\frac{k}{2}\right)^2 + \frac{(1 - k)^2(7 + 8k)}{14(2 - k)(1 + k)}}$$

(12)
Note that for small $k < 1/8$ the exact expressions (7), (9), and (10) are still useful as the approximation gives values slightly larger than unity for $J_\infty$.

The occurrence of the maximum rate of new infections (8) at positive values of the reduced peak time $\tau_{\text{max}} > 0$ requires values of $k < 1 - 2\eta$. In this case the reduced peak time is well approximated by

$$\tau_{\text{max}} \simeq \frac{1}{c_3} \text{artanh} \left[ \frac{2c_3}{c_1 + 2c_0 J_0 - \eta} \right],$$

with $c_0 = \eta(1 - \eta)$ and $c_1 = 1 - k - 2\eta$,

$$c_2 = \frac{J_{\text{max}} - c_0 - c_1 (J_0 - \eta)}{(J_0 - \eta)^2},$$

$$c_3 = \sqrt{\left( \frac{c_1}{2} \right)^2 - c_0 c_2}. \quad (14)$$

The reduced time dependence of the rate of new infections is well approximated as

$$j(\tau) \simeq \begin{cases} \left( \frac{\sinh(c_3 \tau_m)}{\sinh(c_3 \tau + \sqrt{\frac{\ln a_0}{a_0} \sinh[c_2 (\tau_m - \tau)]}} \right)^2 & \text{for } \tau \leq \tau_m \\ \frac{e^{d_1 (\tau_m - \tau)}}{1 + d_1 (\tau_m - \tau)} & \text{for } \tau \geq \tau_m \end{cases}$$

with $d_1 = J_\infty - (1 - k)$.

For a stationary infection rate $a_0$ the corresponding real peak time is given by

$$t_{\text{peak}} = t_0 + \frac{\tau_{\text{max}}}{a_0}.$$  \hspace{1cm} (16)

Likewise, the early asymptotic reduced time behavior is well approximated by

$$j_{\text{early}}(\tau \ll \tau_{\text{max}}) \simeq A e^{(1-k)\tau},$$

corresponding to the early asymptotic real time behavior

$$j_{\text{early}}(t) = a(t) j_{\text{early}}(\tau(t)).$$  \hspace{1cm} (18)

In the considered case of a stationary infection rate Eq. (18) reduces to

$$j_{\text{early}}(t) = a_0 j(\tau = a_0 (t - t_0)) = A a_0 e^{(1-k) a_0 (t-t_0)},$$

implying for the early doubling time defined by $j_{\text{early}}(t + t_2) = 2 j_{\text{early}}(t)$ that

$$t_2 = \frac{\ln 2}{a_0 (1-k)} = \frac{\ln 2}{a_0 - \mu_0},$$

where we inserted $k = \mu_0/a_0$ in the case of stationary infection and recovery rates. Equation (20) will be used in the following two sections in two different ways.

The maximum 7-day incidence value per $10^5$ persons is calculated by integrating

$$\text{SDI} = 7 \times 10^5 \int_{t_{\text{max}}-3.5}^{t_{\text{max}}+3.5} dt \dot{J}(t).$$  \hspace{1cm} (21)

It is only slightly smaller than the estimate $\text{SDI} \simeq 7 \times 10^5 J_{\text{max}}$ from the maximum rate.

The late at times after the maximum half-decay time is given by

$$t_{1/2} = \frac{\ln 2}{a_0 d_1} = \frac{0.693}{a_0 (J_\infty - (1-k))}.$$  \hspace{1cm} (22)

III. CONSEQUENCES OF EARLY 2-DAY DOUBLING TIME

For the omicron mutant the early doubling time of $t_{2,\text{omicron}} = 3$ days has been reported in South Africa, Great Britain and Denmark. Adopting this value for all countries considered then provides according to Eq. (20) for the omicron mutant the relation

$$a_{0,\text{omicron}} = \frac{\ln 2}{3(1 - k_{\text{omicron}})} = \frac{0.231}{1 - k_{\text{omicron}}} \text{ days}^{-1} \quad (23)$$

throughout. Using this relation in all results of the last section to eliminate $a_0$ we find that all quantities of interest are solely determined by the parameter $k$. Particularly for the peak time (16) we obtain

$$t_{\text{peak,omicron}} = t_0 + 4.328 \tau_{\text{max}} (1 - k_{\text{omicron}}).$$

whereas the maximum rate of new infections

$$j_{\text{max,omicron}}(k) = a_{0,\text{omicron}} j_{\text{max}}(k) = \frac{0.231 f_{\text{max}}(k)}{1 - k_{\text{omicron}}} \approx \frac{0.0165 (1 - k_{\text{omicron}})(7 + 8k)}{(2 - k_{\text{omicron}})(1 + k_{\text{omicron}})},$$

Likewise the real time dependence of the rate of new infections with Eq. (14) is given by

$$j_{\text{omicron}}(t) = \frac{0.231 j(0.231 (t-t_0))}{1 - k_{\text{omicron}}}.$$  \hspace{1cm} (26)

In Fig. 1 we display the resulting dependence of $J_\infty$, $t_{\text{max}} - t_0 = t_{\text{max}}/a_{0,\text{omicron}}$ and $J_{\text{max}} = a_{0,\text{omicron}} j_{\text{max}}$ as a function of the parameter $k_{\text{omicron}} \in [0, 1]$. It can be seen that $J_\infty$ and $J_{\text{max}}$ decrease with increasing values of $k$ almost independent of the initial fraction $\eta$ of infected persons, except at very large values of $k_{\text{omicron}}$ close to unity. Obviously, for comparatively small values of the total number of infected persons $J_\infty$ and the maximum rate of newly infected persons $J_{\text{max}}$ large values of the ratio $k$ are required. Alternatively, the reduced time of maximum $\tau_{\text{max}}$ decreases with increasing values of $k$ as long as $k$ is much smaller than $1 - 2\eta$.

IV. OMICRON FORECAST IN INDIVIDUAL COUNTRIES

With the earlier inferred parameter values $a_0^0$ and $k_0$ for the second wave caused by the $\beta$-mutant we calculate the second wave doubling time $t_2^0$ for the countries listed in Table...
TABLE I. Second wave parameters $a_0^\beta$ in days$^{-1}$, $k_\beta$, initial fraction $\eta_\beta$, and the inferred second doubling time $t_2^\beta$ in days. For the omicron mutant in all countries we adopt $t_2^\text{omicron}$ to calculate the ratio of the two doubling times $r = t_2^/t_2^\text{omicron}$.

<table>
<thead>
<tr>
<th>country</th>
<th>$a_0^\beta$</th>
<th>$k_\beta$</th>
<th>$\eta_\beta$</th>
<th>$t_2^\beta$</th>
<th>$t_2^\text{omicron}$</th>
<th>$r = t_2^/t_2^\text{omicron}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>ITA</td>
<td>0.13</td>
<td>0.823</td>
<td>1.1 $\times 10^{-4}$</td>
<td>30.1</td>
<td>3.0</td>
<td>10.03</td>
</tr>
<tr>
<td>AUT</td>
<td>0.43</td>
<td>0.898</td>
<td>1.8 $\times 10^{-5}$</td>
<td>15.8</td>
<td>3.0</td>
<td>5.27</td>
</tr>
<tr>
<td>DNK</td>
<td>2.48</td>
<td>0.972</td>
<td>3.6 $\times 10^{-5}$</td>
<td>10.0</td>
<td>3.0</td>
<td>3.33</td>
</tr>
<tr>
<td>DEU</td>
<td>0.45</td>
<td>0.907</td>
<td>1.1 $\times 10^{-5}$</td>
<td>16.6</td>
<td>3.0</td>
<td>5.53</td>
</tr>
<tr>
<td>CHE</td>
<td>0.44</td>
<td>0.892</td>
<td>2.2 $\times 10^{-5}$</td>
<td>14.6</td>
<td>3.0</td>
<td>4.87</td>
</tr>
<tr>
<td>GBR</td>
<td>0.44</td>
<td>0.874</td>
<td>4.6 $\times 10^{-5}$</td>
<td>12.5</td>
<td>3.0</td>
<td>4.17</td>
</tr>
<tr>
<td>FRA</td>
<td>0.17</td>
<td>0.868</td>
<td>1.0 $\times 10^{-4}$</td>
<td>30.9</td>
<td>3.0</td>
<td>10.30</td>
</tr>
<tr>
<td>BEL</td>
<td>0.53</td>
<td>0.893</td>
<td>1.8 $\times 10^{-4}$</td>
<td>12.2</td>
<td>3.0</td>
<td>4.07</td>
</tr>
<tr>
<td>NLD</td>
<td>0.37</td>
<td>0.926</td>
<td>3.4 $\times 10^{-5}$</td>
<td>25.3</td>
<td>3.0</td>
<td>8.43</td>
</tr>
<tr>
<td>RUS</td>
<td>0.03</td>
<td>0.801</td>
<td>6.9 $\times 10^{-3}$</td>
<td>116.1</td>
<td>3.0</td>
<td>38.70</td>
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<tr>
<td>SWE</td>
<td>0.58</td>
<td>0.919</td>
<td>8.0 $\times 10^{-9}$</td>
<td>14.8</td>
<td>3.0</td>
<td>4.93</td>
</tr>
<tr>
<td>USA</td>
<td>0.22</td>
<td>0.868</td>
<td>9.5 $\times 10^{-4}$</td>
<td>23.9</td>
<td>3.0</td>
<td>7.97</td>
</tr>
</tbody>
</table>

yielding readily the relation

$$a_0^\text{omicron} (1 - k_\text{omicron}) = r a_0^\beta (1 - k_\beta).$$
(28)

For each country we then consider 3 possible omicron scenarios:

(1) the optimistic case with $k_\text{omicron} = k_\beta$ so that the increase in the ratio $r$ is solely due to the increase in the stationary infection rate

$$a_0^\text{omicron} = r a_0^\beta$$
(29)

As noted earlier the larger the value of $k_\text{omicron}$ the smaller the total cumulative number of infections $J_\infty$ and the maximum rate of new infections $J_{\text{max}}$ will be. This justifies the classification of this case as optimistic.

(2) the pessimistic scenario with $a_0^\text{omicron, pessim} = a_0^\beta$ so that the increase in the ratio $r$ is solely due to the decrease in the ratio $k$

$$k_\text{omicron, pessim} = 1 - r(1 - k_\beta)$$
(30)

Clearly, with these small values of $k_\text{omicron}$, the resulting total cumulative number of infections $J_\infty$ and the maximum rate of new infections $J_{\text{max}}$ will be highest, justifying the classification of this case as pessimistic. In four countries (ITA, FRA, RUS, USA) the resulting $k_\text{omicron, pessim}$ is negative which cannot be. In these cases we use $k_\text{omicron, pessim} = 0$ and $a_0^\text{omicron, pessim} = 0.231$.

(3) the intermediate case with

$$a_0^\text{omicron, interm} = \frac{r}{2} a_0^\beta,$$
(31)

where half of the increase in the ratio $r$ stems from the increase in the stationary infection rate. Then as a consequence

$$k_\text{omicron, interm} = 2k_\beta - 1$$
(32)
of the three regimes for the 12 countries, and Fig. 3 shows Table III. Forecast of the omicron mutant for the pessimistic case, i.e., $a_0 = a_0^{\text{omicron, pess}}$ and $k = k^{\text{omicron, pess}}$, and initial fraction $\eta = \eta_j$ from Tab. I for this table. Columns list the final cumulative fraction $J_{\infty}$ of infected persons, the maximum (dimensionless) rate $J_{\max}$ of new infections, the cumulative fraction $J_0$ of infected persons at peak time, the reduced peak time $\tau_{\max}$, the peak time $t_{\max}$, and the SDI, the maximum 7-day incidence per $10^5$ persons. Country names are abbreviated by their $\alpha_j$ codes.

<table>
<thead>
<tr>
<th>Country</th>
<th>$\alpha_j$</th>
<th>$a_0$</th>
<th>$k$</th>
<th>$J_{\infty}$</th>
<th>$J_{\max}$</th>
<th>$J_0$</th>
<th>$\tau_{\max}$</th>
<th>$t_{\max}$</th>
<th>SDI</th>
</tr>
</thead>
<tbody>
<tr>
<td>ITA</td>
<td>1.304</td>
<td>0.823</td>
<td>0.33</td>
<td>0.0139</td>
<td>0.018</td>
<td>0.160</td>
<td>35.6</td>
<td>27 days</td>
<td>12725</td>
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<tr>
<td>AUT</td>
<td>2.265</td>
<td>0.898</td>
<td>0.20</td>
<td>0.0049</td>
<td>0.011</td>
<td>0.097</td>
<td>68.9</td>
<td>30 days</td>
<td>7716</td>
</tr>
<tr>
<td>DNK</td>
<td>8.267</td>
<td>0.972</td>
<td>0.06</td>
<td>0.0004</td>
<td>0.004</td>
<td>0.028</td>
<td>130.1</td>
<td>16 days</td>
<td>2424</td>
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<tr>
<td>DEU</td>
<td>2.490</td>
<td>0.907</td>
<td>0.18</td>
<td>0.0041</td>
<td>0.010</td>
<td>0.089</td>
<td>79.0</td>
<td>32 days</td>
<td>7090</td>
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<td>CHE</td>
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<td>0.892</td>
<td>0.21</td>
<td>0.0054</td>
<td>0.012</td>
<td>0.102</td>
<td>64.0</td>
<td>30 days</td>
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<tr>
<td>GBR</td>
<td>1.833</td>
<td>0.874</td>
<td>0.24</td>
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<td>0.013</td>
<td>0.118</td>
<td>51.3</td>
<td>28 days</td>
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<td>0.014</td>
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<td>0.21</td>
<td>0.0055</td>
<td>0.012</td>
<td>0.101</td>
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<tr>
<td>RUS</td>
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<td>0.801</td>
<td>0.39</td>
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<td>9 days</td>
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<td>0.919</td>
<td>0.16</td>
<td>0.0031</td>
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<td>0.078</td>
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<td>61 days</td>
<td>6216</td>
</tr>
<tr>
<td>USA</td>
<td>1.753</td>
<td>0.868</td>
<td>0.26</td>
<td>0.0087</td>
<td>0.015</td>
<td>0.122</td>
<td>26.0</td>
<td>15 days</td>
<td>10650</td>
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</tbody>
</table>

Table II. Forecast of the omicron mutant for the optimistic case, i.e., $a_0 = a_0^{\text{omicron, opti}}$ and $k = k^{\text{omicron, opti}}$. The corresponding predicted maximum 7-day incidence values per $10^5$ persons (SDI) are 2424, 4462 and 7148, respectively. Presently on January 10, 2022 the well-monitored data of Denmark[13] indicate that the SDI has saturated at its maximum value at 2478 which is in excellent agreement with our predicted value in the optimistic case. Although preliminary this outstanding agreement is definitely encouraging and argues in favour of the optimistic scenario. Regarding Germany we predict peak times of the omicron wave ranging from 32 to 38 and 45 days after the start of the omicron wave in the optimistic, intermediate and pessimistic scenario, respectively, with corresponding maximum SDI values of 7090, 13263 and 2891, respectively. Adopting Jan 1st, 2022 as the starting date our predictions implies that the maximum of the omicron wave is reached between Feb 1 and Feb 15, 2022. In the optimistic case the total cumulative number of omicron infections will be 0.180 but can go up high to 0.824 in the pessimistic case. The late half decay times are 3.2 to 3.5 and 5.2 days in the optimistic, intermediate and pessimistic case, respectively.

Table III shows that the peak times of the omicron wave range from 30 to 36 and 45 days after the start of the omicron wave in the optimistic, intermediate and pessimistic scenario, respectively. It is obvious from these three tables that in European countries, apart from Russia with limited data reliability, Denmark has the shortest peak time of the omicron wave ranging from 16 to 22 and 27 days after the start of the omicron wave in the optimistic, intermediate and pessimistic scenario, respectively. The corresponding predicted maximum 7-day

Table IV. Forecast of the omicron mutant for the intermediate case, i.e., $a_0 = a_0^{\text{omicron, inter}}$ and $k = k^{\text{omicron, inter}}$. Table IV visualizes the relationship between $a_0$ and $k$ and the location of the three regimes for the 12 countries, and Fig. 3 shows the time dependence of $J(t)$ and cumulative fraction $J_{\infty}$ of infected persons for all 12 countries.

In Tables II, III and IV we calculate the forecast for the omicron mutant for these three scenarios, respectively. Figure 2 visualizes the relationship between $a_0$ and $k$ and the location of the three regimes for the 12 countries, and Fig. 3 shows the time dependence of $J(t)$ and cumulative fraction $J_{\infty}$ of infected persons for all 12 countries.

It is obvious from these three tables that in European countries, apart from Russia with limited data reliability, Denmark has the shortest peak time of the omicron wave ranging from 16 to 22 and 27 days after the start of the omicron wave in the optimistic, intermediate and pessimistic scenario, respectively. The corresponding predicted maximum 7-day

V. MEDICAL CONSEQUENCES FOR GERMANY

A. Tolerable maximum 7-day incidence value

We have argued earlier[14] that the German health system can cope with maximum SDI values of $280/(hm)$ without any
triation decisions, where \( m \) in months denotes the average duration of intensive care with access to breathing apparatus for seriously infected persons, and \( h \) in units of percent indicates the percentage of people seriously infected needing access to breathing apparatus in hospitals. For the earlier \( \alpha \) and \( \beta \) mutants the value of \( h = 1 \) has been reasonable. Fortunately, for the omicron variant from studies in South Africa\textsuperscript{13} and Great Britain\textsuperscript{16} substantial 70–90 percent reductions as compared to earlier mutants in Covid-19 hospitalization have been reported. This strong reduction is predominantly caused by the high percentage of persons with boosted vaccination.

We therefore adopt here the value of \( h_{\text{omicron}} = 0.1 \), i.e. only one out of 1000 new infections with the omicron mutant needs to be hospitalised. Consequently, the German health system can cope with maximum omicron SDI value of 2800/\( m \) which is about a factor 2.5 smaller than the maximum omicron SDI value 7090 in the optimistic case. By either (1) reducing the duration of intensive care during this period of maximum to \( m = 0.5 \), and/or (2) by making use of the nonuniform spread of the omicron wave across Germany, appearing first in the northern states and considerably later in the southern and eastern states, combined with mutual help in hospital capacities, it seems that the German health system can cope with the omicron wave avoiding triage decisions.

B. Fatality rates and total number of fatalities

As before\textsuperscript{10,14} we assume that every second hospitalised person eventually dies from the omicron virus so that the omicron mortality rate is \( f_{\text{omicron}} = 0.5 h_{\text{omicron}} = 5 \times 10^{-4} \) which is one order of magnitude smaller than the mortality rates of the earlier mutants. Consequently, the total fatality rate is given by \( D_\infty = f_{\text{omicron}} D_\infty N \), where \( N = 82.7 \) million denotes the German population. Likewise the maximum death rate is \( d_{\text{max}} = f_{\text{omicron}} N d_{\text{max}} \). In the optimistic scenario one obtains \( D_\infty = 7445 \) and \( d_{\text{max}} = 418 \) per day which are about one order of magnitude smaller than the beta fatality rate and total number of fatalities of the second wave\textsuperscript{10}. The main reason for these comparatively small numbers is the order of magnitude smaller hospitalization rate of the omicron mutant compared to the earlier more deadly mutants.

However, in the less likely pessimistic scenario the fatality numbers increase by a factor 4.5 to 5 to \( D_\infty = 33576 \) and \( d_{\text{max}} = 1708 \) which are about half of the fatality values of the second wave.

VI. SUMMARY AND CONCLUSIONS

Adopting an early doubling time of three days for the rate of new infections with the omicron mutant the temporal evolution of the omicron wave in different countries is predicted. The predictions are based on the susceptible-infectious-recovered/removed (SIR) epidemic compartment model with a constant stationary ratio \( k = \mu(t)/\alpha(t) \) between the infection \( (\alpha(t)) \) and recovery \( (\mu(t)) \) rate. The fixed early doubling time then uniquely relates the initial infection rate \( a_0 \) to the ratio \( k \), which therefore determines the full temporal evolution of the omicron waves.

As all considered countries have been exposed to earlier waves of the Covid-19 virus we relate the parameters \( a_0 \) and \( k \) to those of the well-studied second wave. In the optimistic case we assume that the decrease in the early doubling time of the omicron mutant as compared to the beta mutant is solely due to a corresponding increase in the initial infection rate \( a_0 \) whereas the ratio \( k \) is the same as for the beta mutant. In the pessimistic case we assume that the decrease in the early doubling times is fully caused by a corresponding decrease in the ratio \( k \) whereas the initial infection rate is the same as for the beta mutant. In the intermediate scenario half of the decrease in the early doubling time is assigned to a corresponding increase of the initial infection rate \( a_0 \) whereas the ratio \( k \) is the same as for the beta mutant. In the intermediate scenario half of the decrease in the early doubling time is assigned to a corresponding increase of the initial infection rate and a corresponding decrease of the ratio \( k \). For 12 countries these three scenarios (optimistic, pessimistic, intermediate) are considered and the resulting pandemic parameters are calculated. These include the total number of infected persons, the maximum rate of new infections, the peak time and the maximum 7-day incidence per 100000 persons.

Among the considered European countries Denmark has the smallest omicron peak time and the recently observed saturation of the 7-day incidence value at 2478 is in excellent agreement with our prediction in the optimistic case. For Germany we predict peak times of the omicron wave ranging from 32 to 38 and 45 days after the start of the omicron wave in the optimistic, intermediate and pessimistic scenario, respectively, with corresponding maximum SDI values of 7090, 13263 and 28911, respectively. Adopting Jan 1st, 2022 as the starting date our predictions implies that the maximum of the omicron wave is reached between Feb 1 and Feb 15, 2022. In the optimistic case the total cumulative number of omicron infections will be 0.180 but can go up high to 0.812 in the pessimistic case. The late half decay times are 3.2 to 3.5 and 5.2 days in the optimistic, intermediate and pessimistic case.

\[ D_\infty = 7445 \]
\[ d_{\text{max}} = 418 \]
\[ D_\infty = 33576 \]
\[ d_{\text{max}} = 1708 \]
FIG. 3. Time dependence of the daily rate of newly infected persons, $\dot{J}(t)$, as well as the cumulative fraction of infected persons, $J(t)$, for all 12 countries.

respectively.

Rather similar values are predicted for Switzerland. Here the peak times of the omicron wave range from 30 to 36 and 42 days after the start and the corresponding maximum SDI values are 8148, 15060 and 29259, respectively. Here with the same starting date the maximum of the omicron wave is reached between Jan 31 and Feb 13, 2022. Here, in the optimistic case the total cumulative number of omicron infections
will be 0.208 but can go up high to 0.824 in the pessimistic case. The late half decay times are 3.2 to 3.6 and 5.3 days in the optimistic, intermediate and pessimistic case, respectively.

Adopting an order of magnitude smaller omicron hospitalization rate thanks to the high percentage of vaccinated and booster population we conclude that the German health system can cope with maximum omicron SDI value of 2800 which is about a factor 2.5 smaller than the maximum omicron SDI value 7090 in the optimistic case. By either reducing the duration of intensive care during this period of maximum, and/or by making use of the nonuniform spread of the omicron wave across Germany, it seems that the German health system can barely cope with the omicron wave avoiding triage decisions.

The reduced omicron hospitalization rate also causes significantly smaller mortality rates compared to the earlier mutants in Germany. In the optimistic scenario one obtains for the total number of fatalities $D_\infty = 7445$ and for the maximum death rate $d_{\text{max}} = 418$ per day which are about one order of magnitude smaller than the beta fatality rate and total number. In the less likely pessimistic scenario these numbers increase by a factor 4.5.

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